

Decentralized Failure Diagnosis of Stochastic Discrete Event Systems *

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Abstract: Recently, the diagnosability of *stochastic discrete event systems* (SDESs) was investigated in the literature, and, the failure diagnosis considered was *centralized*. In this paper, we propose an approach to *decentralized* failure diagnosis of SDESs, where the stochastic system uses multiple local diagnosers to detect failures and each local diagnoser possesses its own information. In a way, the centralized failure diagnosis of SDESs can be viewed as a special case of the decentralized failure diagnosis presented in this paper with only one projection. The main contributions are as follows: (1) We formalize the notion of codiagnosability for stochastic automata, which means that a failure can be detected by at least one local stochastic diagnoser within a finite delay. (2) We construct a codiagnoser from a given stochastic automaton with multiple projections, and the codiagnoser associated with the local diagnosers is used to test codiagnosability condition of SDESs. (3) We deal with a number of basic properties of the codiagnoser. In particular, a necessary and sufficient condition for the codiagnosability of SDESs is presented. (4) We give a computing method in detail to check whether codiagnosability is violated. And (5) some examples are described to illustrate the applications of the codiagnosability and its computing method.

Index Terms: Discrete event systems, failure diagnosis, decentralized diagnosis, stochastic automata, codiagnosability.

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I. Introduction

A *discrete event system* (DES) is a dynamical system whose state space is discrete and whose states can only change when certain events occur [1, 2], which has been successfully applied to provide a formal treatment of many technological and engineering systems [1]. In order to guarantee performance to a reliable system, the control engineers should design a system that runs safely within its normal boundaries. Therefore, failure diagnoses in DESs, which are to detect and isolate the unobservable fault events occurring in a system within a finite delay, are of practical and theoretical importance, and have received considerable attention in recent years [1, 3-34].

In the past a long time, most of the research works on failure diagnosis of DESs in the literature focused on centralized failure diagnosis usually [3, 5, 9-17, 19, 20, 23-27, 29, 31-34]. Many large complex systems, however, are physically distributed systems in nature [6, 7, 21], where information diagnosed is decentralized and there are several local sites, in which sensors report their data and diagnosers run at each site processing the local observation. Therefore, in recent years, more and more research works have devoted to decentralized failure diagnosis [4, 6-8, 18, 21, 22, 28, 30].

As we know, the classical DES models cannot distinguish between strings or states that are highly probable and those that are less probable, and the notion that a failure can be diagnosed after a finite delay is “all-or-nothing” [31]. Stochastic automata, as a natural generalization for deterministic automata of different types, are a more precise formulation of the general DES models, in which a probabilistic structure is appended to estimate the likelihood of specific events occurring [31]. An introduction to the theory of stochastic automata can be found in [2].

More recently, by generalizing the diagnosability of classical DESs [25, 26] to the setting of *stochastic discrete event systems* (SDESs), the diagnosability of SDESs was interestingly dealt with by J. Lunze and J. Schröder [16], D. Thorsley and D. Teneketzis [31]. In [16], the diagnostic problem was transformed into an observation problem, and the diagnosability was obtained by an extension of an observation algorithm. In [31], the notions of A- and AA-diagnosability for stochastic automata were defined, which were weaker than those for classical automata introduced by Sampath *et al* [25, 26], and they presented a necessary and sufficient condition for the diagnosability of SDESs. However, the failure diagnosis considered in SDESs was *centralized*. Therefore, motivated by the importance of decentralized failure diagnosis, our goal is to deal with the decentralized failure diagnosis for SDESs.

In this paper, we formalize the approach to decentralized failure diagnosis in SDESs by introducing the notion of codiagnosability. The centralized failure diagnosis in SDESs [16, 31] can be viewed as a special case of the decentralized failure diagnosis presented in this paper with only one projection. Roughly speaking, a language generated by a stochastic

automaton is said to be codiagnosable under some local projections if, after a failure event occurs, there exists at least one local site such that the probability of non-diagnosing failure is sufficiently small within a finite delay. By constructing a codiagnoser from a given stochastic automaton with multiple projections, we can use the codiagnoser associated with the local diagnosers to test codiagnosability condition of SDESs. As well, a number of basic properties of the codiagnoser is investigated. In particular, a necessary and sufficient condition for the codiagnosability of SDESs is presented, which generalizes the result of classical DESs dealt with by W. Qiu and R. Kumar [21]. Furthermore, we propose a computing method in detail to check whether codiagnosability is violated. Finally, some examples are described to illustrate the applications of the codiagnosability and its computing method.

This paper is organized as follows. Section II serves to recall some related concepts and notations concerning failure diagnosis of DESs and SDESs. In Section III, we introduce a definition of the codiagnosability of SDESs. The codiagnoser used to detect failure in SDESs is constructed. In Section IV, some main properties of codiagnoser are investigated. In particular, a necessary and sufficient condition for the codiagnosability of SDESs is presented. As well, we give a computing method in detail to check whether codiagnosability is violated, according to the codiagnoser and the local stochastic diagnosers. Furthermore, some examples are provided to illustrate the condition of the codiagnosability for SDESs. Finally, in Section V, we summarize the main results of the paper and address some related issues.

II. Notations and preliminaries

In this section, we present some preliminaries concerning stochastic automata and centralized failure diagnosis of SDESs. For more details on SDESs, we can refer to [2,16,31].

A. Stochastic Automata

A stochastic automaton is a finite state machine (FSM) with a probabilistic structure.

Definition 1[31]: A *stochastic automaton* is a type of systems with a quadruple

$$G = (Q, \Sigma, \eta, q_0), \quad (1)$$

where Q is a finite state space; $q_0 \in Q$ is the initial state; Σ is a finite set of events; $\eta : Q \times \Sigma \times Q \rightarrow [0, 1]$ is a state transition function: for any $q, q' \in Q$ and any $\sigma \in \Sigma$, $\eta(q, \sigma, q')$ represents the probability that a certain event σ will occur, together with transferring the state of the machine from a given state q to the specified state q' . For example, $\eta(q, \sigma, q') = 0.7$ means that, if the machine is in state q , then with probability 0.7 event σ will occur, together with transferring to state q' .

For the sake of simplicity, we assume like [31] that, for a given state $q \in Q$ and a given event $\sigma \in \Sigma$, there exists at most one state $q' \in Q$ such that $\eta(q, \sigma, q') > 0$. Therefore, we can sometimes use the symbol $\eta(q, \sigma)$ instead of $\eta(q, \sigma, q')$ hereafter. Moreover, we recursively define

$$Pr(\sigma | q) = \eta(q, \sigma), \quad Pr(s\sigma | q) = Pr(s | q)\eta(q', \sigma), \quad (2)$$

where $\sigma \in \Sigma$, $s \in \Sigma^*$ and $\eta(q, s, q') > 0$. Intuitively, $Pr(\sigma | q)$ or $Pr(s\sigma | q)$ represents the probability of event σ or string $s\sigma$ being the next event or string when the system is in state q . We can simply denote them by $Pr(\sigma)$ and $Pr(s\sigma)$, respectively, if no confusion results.

Some events in Σ occurring can be observed by the sensors, while the rest are unobservable. That is, $\Sigma = \Sigma_o \cup \Sigma_{uo}$, where Σ_o represents the set of observable events and Σ_{uo} the set of unobservable events. Let $\Sigma_f \subseteq \Sigma$ denote the set of failure events which are to be diagnosed. Without loss of generality, we can assume that $\Sigma_f \subseteq \Sigma_{uo}$, as [24, 28, 29, 34]. And Σ_f is partitioned into different failure types

$$\Sigma_f = \Sigma_{f_1} \cup \Sigma_{f_2} \cup \dots \cup \Sigma_{f_m}.$$

If a failure event $\sigma \in \Sigma_{f_i}$ occurs, we will say that a failure type F_i has occurred.

The language generated by a stochastic automaton G , denoted by $L(G)$, or L for simplicity, is the set of all finite strings with positive probability. That is,

$$L = \{s \in \Sigma^* : (\exists q \in Q)\eta(q_0, s, q) > 0\}. \quad (3)$$

A trace $s \in L$ is called to be a deadlocking trace if no further continuations exist after it is in L , i.e., $\{s\}\Sigma^* \cap L = \{s\}$. Without loss of generality, we assume that L is deadlock-free. Otherwise, we can extend each deadlocking trace by an unbounded sequence of a newly added event that is unobservable to all diagnosers. This will make the language deadlock-free without altering any properties of diagnosability [12, 21].

When a system execution is observed by an observation, events executed by the system are filtered and the unobservable events are erased by a projection.

Definition 2: A *projection* $P : \Sigma^* \rightarrow \Sigma_o^*$ is defined: $P(\epsilon) = \epsilon$, and for any $\sigma \in \Sigma$, $s \in \Sigma^*$, $P(s\sigma) = P(s)P(\sigma)$, where

$$P(\sigma) = \begin{cases} \sigma, & \text{if } \sigma \in \Sigma_o, \\ \epsilon, & \text{if } \sigma \in \Sigma_{uo}. \end{cases} \quad (4)$$

And the inverse projection of string $y \in \Sigma_o^*$ is defined as

$$P^{-1}(y) = \{s \in L : P(s) = y\}. \quad (5)$$

We further need some notations. For string $s \in \Sigma^*$, \bar{s} and s_f denote the prefix-closure of s and the final event of string s , respectively. We define

$$L/s = \{t \in \Sigma^* : st \in L\}, \quad (6)$$

$$\Psi(\Sigma_{f_i}) = \{s\sigma_f \in L : \sigma_f \in \Sigma_{f_i}\}, \quad (7)$$

$$L(G, q) = \left\{s \in \Sigma^* : (\exists q' \in Q) \eta(q, s, q') > 0\right\}. \quad (8)$$

Intuitively, L/s represents the set of continuations of the string s , $\Psi(\Sigma_{f_i})$ denotes the set of all traces of L that end in a failure event belonging to the class Σ_{f_i} , and $L(G, q)$ denotes the set of all traces that originate from state $q \in Q$.

B. Centralized Failure Diagnosis of SDESSs

Before discussing the decentralized failure diagnosis of SDESSs, we recall the centralized failure diagnoses of SDESSs investigated in [31].

Definition 3 [31]: Let L be a language generated by a stochastic automaton $G = (Q, \Sigma, \eta, q_0)$ and let $P : \Sigma^* \rightarrow \Sigma_o^*$ be a projection. L is said to be *A-diagnosable with respect to P* if

$$\begin{aligned} &(\forall \epsilon > 0)(\exists n_0 \in \mathbf{N})(\forall s \in \Psi(\Sigma_{f_i}) \wedge n \geq n_0) \\ &\{Pr(t : D(st) = 0 \mid t \in L/s \wedge \|t\| = n) < \epsilon\} \end{aligned} \quad (9)$$

where the diagnosability condition function $D : \Sigma^* \rightarrow \{0, 1\}$ is defined as follows:

$$D(st) = \begin{cases} 1, & \text{if } \omega \in P^{-1}[P(st)] \Rightarrow \Sigma_{f_i} \in \omega, \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

Roughly speaking, L being *A-diagnosable* means that after a failure occurs, the probability that a string cannot be detected is sufficiently small within a finite delay. For simplicity, we will call it *diagnosable* instead of *A-diagnosable*.

The stochastic diagnoser G_d constructed in [31] is as follows:

$$G_d = (Q_d, \Sigma_o, \delta_d, \chi_0, \Phi, \phi_0), \quad (11)$$

where Q_d is the set of states of the diagnoser with initial state $\chi_0 = \{(q_0, N)\}$, Σ_o is the set of observable events, δ_d is the transition function of the diagnoser, Φ is the set of probability transition matrices, and ϕ_0 is the initial probability mass function on χ_0 .

Some concepts concerning finite state Markov chain are used to derive the results of [31], and we briefly recall them. Suppose that x and y are two states of a Markov chain. The symbol ρ_{xy} represents the probability that if the Markov chain is in state x , it will visit state y at some point in the future. For a state x , if $\rho_{xx} = 1$, then x is called a *recurrent state*. Otherwise, if $\rho_{xx} < 1$, then x is called a *transient state*.

We now quote the basic properties related to transient or recurrent states from [31].

Lemma 1 (Lemma 1 in [31]): Let Γ be the set of transient states of a Markov chain and let x be an arbitrary state of the chain. Then for any $t \in L(G, x)$ and any $\epsilon > 0$, there exists $n \in \mathbf{N}$ such that

$$Pr(t : \delta(x, t) \in \Gamma \mid t \in L(G, x) \wedge \|t\| = n) < \epsilon. \quad (12)$$

Lemma 2 (Property 4 in [31]): All components reachable from a recurrent state bearing the label F_i in an F_i -uncertain element of Q_d are contained in F_i -uncertain elements.

Using the stochastic diagnoser G_d , Thorsley and Teneketzis [31] presented a necessary and sufficient condition for diagnosability as follows:

Lemma 3 (Theorem 3 in [31]): A language L generated by a stochastic automaton G is diagnosable, if and only if, every logical element of its diagnoser G_d containing a recurrent component bearing the label F is F -certain.

III. Codiagnosability and Codiagnoser for SDESs

In order to illustrate the solution to the decentralized failure diagnosis problem, we make the following assumptions about the stochastic automaton $G = (Q, \Sigma, \eta, q_0)$ as [31]:

(A1): The language $L = L(G)$ is live. That is to say, for any $q \in Q$,

$$\sum_{q' \in Q} \sum_{\sigma \in \Sigma} \eta(q, \sigma, q') = 1. \quad (13)$$

(A2): There does not exist any cycle of unobservable events, i.e.,

$$(\exists n_0 \in \mathbf{N})(\forall ust \in L)[(s \in \Sigma_{uo}^* \Rightarrow (\| s \| \leq n_0))].$$

Intuitively, assumption (A1) means that the sum of the probability of all transitions from each state is equal to one, which indicates that transitions will continue to occur in any state. Assumption (A2) ensures that G does not generate arbitrarily long sequences of unobservable events, because failure diagnosis is based on observable transitions of the system.

A. Approaches to Defining Codiagnosability for SDESs

In this subsection, we consider decentralized failure diagnosis where there are m local diagnosers to detect system G . The m local diagnosers are assumed to be independent, namely, without communicating their observations each other [15, 24-26, 31]. From the m local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$ defined as Definition 2, where $i = 1, 2, \dots, m$, we can obtain the global projection $P : \Sigma^* \rightarrow \Sigma_o^*$, in which

$$\Sigma_o = \Sigma_{o,1} \cup \Sigma_{o,2} \cup \dots \cup \Sigma_{o,m}. \quad (14)$$

Now let us give the definition of codiagnosability for SDESs.

Definition 4: Let $G = (Q, \Sigma, \eta, q_0)$ be a stochastic automaton, $L = L(G)$. Assume there are m local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2, \dots, m$. Then L is said to be

codiagnosable with respect to $\{P_i\}$ if

$$(\forall \epsilon > 0)(\exists n_i \in \mathbf{N})(\forall s \in \Psi(\Sigma_{f_i}) \wedge n \geq n_i)(\exists j \in \{1, 2, \dots, m\}) \{Pr(t : D_j(st) = 0 \mid t \in L/s \wedge \|t\| = n) < \epsilon\} \quad (15)$$

where for each $j \in \{1, 2, \dots, m\}$, the diagnosability condition function $D_j : \Sigma^* \rightarrow \{0, 1\}$ is defined by Definition 3, i.e.,

$$D_j(st) = \begin{cases} 1, & \text{if } \omega \in P_j^{-1}[P_j(st)] \Rightarrow \Sigma_{f_i} \in \omega, \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

Intuitively, L being codiagnosable means that, for any a trace s that ends in a failure event belonging to Σ_{f_i} and for any a sufficiently long continuation t of s , there exists at least one site j such that, the probability that the j th diagnoser cannot detect the failure among the traces indistinguishable from st for site j is sufficiently small within a finite delay.

Remark 1: Comparing with Definition 3, we know that diagnosability of the centralized system [31] can be viewed as a special case of the codiagnosability of the decentralized system with $m = 1$.

Example 1. Consider the stochastic automaton $G = (Q, \Sigma, \eta, q_0)$ described by Fig.1, where $Q = \{q_0, q_1, \dots, q_6\}$, q_0 is the initial state, $\Sigma = \{a, b, c, d, \sigma_{uo}, \sigma_f\}$, and the set of failure events $\Sigma_f = \{\sigma_f\}$. Assume that there are two local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $\Sigma_{o,1} = \{a, b\}$, $\Sigma_{o,2} = \{a, c\}$, $i = 1, 2$.

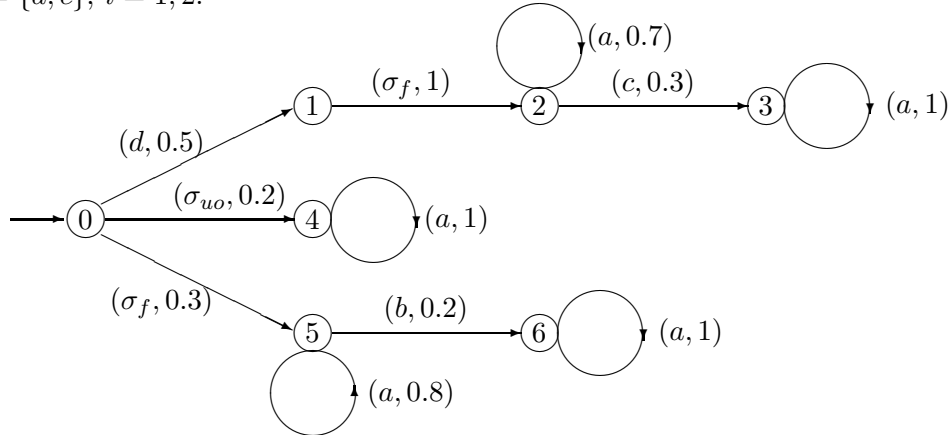


Fig. 1. Stochastic automaton of Example 1.

We assert that the language $L = L(G)$ is codiagnosable. In fact, for any $s \in \Psi(\Sigma_f)$, i.e., $s = \sigma_f$ or $s = d\sigma_f$, we verify its codiagnosability as follows.

Case 1. If $s = \sigma_f$, then for any $t \in L/s$ and $\|t\| = n$, either $t = a^{n-k-1}ba^k$ (where $0 \leq k \leq n-1$) or $t = a^n$, and we can take the first diagnoser to detect the failure.

When $t = a^{n-k-1}ba^k$, we have

$$P_1^{-1}[P_1(st)] = \{\sigma_f a^{n-k-1}ba^k : 0 \leq k \leq n-1\}.$$

Due to $\sigma_f \in \sigma_f a^{n-k-1} b a^k$ for all $k \in [0, n-1]$, we get $D_1(st) = 1$. Therefore, the failure is diagnosed. When $t = a^n$, we have

$$P_1^{-1}[P_1(st)] = \left\{ d\sigma_f a^n, \sigma_f a^n, \sigma_{uo} a^n, d\sigma_f a^{n-k} c a^k : 0 \leq k \leq n-1 \right\}.$$

Due to $\sigma_f \notin \sigma_{uo} a^n$, we obtain $D_1(st) = 0$. In this case, $Pr(t) = Pr(a^n) = 0.8^n$, and with n increasing, the probability that is not diagnosable approaches to zero.

Case 2. If $s = d\sigma_f$, then for any $t \in L/s$ and $\|t\| = n$, either $t = a^{n-k-1} c a^k$ (where $0 \leq k \leq n-1$) or $t = a^n$, and we can take the second diagnoser to detect the failure.

When $t = a^{n-k-1} c a^k$, we have

$$P_2^{-1}[P_2(st)] = \left\{ d\sigma_f a^{n-k-1} c a^k : 0 \leq k \leq n-1 \right\}.$$

Due to $\sigma_f \in d\sigma_f a^{n-k-1} c a^k$ for all $k \in [0, n-1]$, we get $D_2(st) = 1$. Therefore, the failure is diagnosed. When $t = a^n$, we have

$$P_2^{-1}[P_2(st)] = \left\{ d\sigma_f a^n, \sigma_f a^n, \sigma_{uo} a^n, \sigma_f a^{n-k} b a^k : 0 \leq k \leq n-1 \right\}.$$

Due to $\sigma_f \notin \sigma_{uo} a^n$, we obtain $D_2(st) = 0$. In this case, $Pr(t) = Pr(a^n) = 0.7^n$, and with n increasing, the probability that is not diagnosable approaches to zero.

By Definition 4, Case 1 and Case 2 indicate L is codiagnosable. \square

Before constructing the codiagnoser for SDESSs, we first give two propositions for the condition of non-codiagnosability, which can be straight obtained from Definition 4.

Proposition 1: Let $G = (Q, \Sigma, \eta, q_0)$ be a stochastic automaton, $L = L(G)$. Assume there are m local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2, \dots, m$. If there exist $i_0 \in \{1, 2, \dots, m\}$, such that L is diagnosable with respect to P_{i_0} , then L is codiagnosable with respect to $\{P_i\}$.

Proof: If there exists $i_0 \in \{1, 2, \dots, m\}$, such that L is diagnosable with respect to P_{i_0} , then from Definition 3, the following holds:

$$\begin{aligned} & (\forall \epsilon > 0)(\exists n_0 \in \mathbf{N})(\forall s \in \Psi(\Sigma_{f_i}) \wedge n \geq n_0) \\ & \{Pr(t : D_{i_0}(st) = 0 \mid t \in L/s \wedge \|t\| = n) < \epsilon\}. \end{aligned} \quad (17)$$

Therefore, we have

$$\begin{aligned} & (\forall \epsilon > 0)(\exists n_i = n_0)(\forall s \in \Psi(\Sigma_{f_i}) \wedge n \geq n_i)(\exists j = i_0) \\ & \{Pr(t : D_j(st) = 0 \mid t \in L/s \wedge \|t\| = n) < \epsilon\}. \end{aligned} \quad (18)$$

It indicates that L is codiagnosable with respect to $\{P_i : i = 1, 2, \dots, m\}$ by Definition 4. \square

Remark 2: This proposition shows that a system can detect all of the failure strings if a local diagnoser can detect them. However, the inverse proposition does not always hold. That is, there exists the case that a system can still detect all of the failure strings even if all of the local diagnosers cannot detect the failures. Example 2 verifies this view.

Example 2. For the stochastic automaton $G = (Q, \Sigma, \eta, q_0)$ as in Example 1, we have known that the language L is codiagnosable from Example 1. However, in the following we prove that L is not diagnosable with respect to $P_1 : \Sigma^* \rightarrow \Sigma_{o,1}^*$, neither is L diagnosable with respect to $P_2 : \Sigma^* \rightarrow \Sigma_{o,2}^*$, where $\Sigma_{o,1} = \{a, b\}$ and $\Sigma_{o,2} = \{a, c\}$.

In fact, for the first projection $P_1 : \Sigma^* \rightarrow \Sigma_{o,1}^*$, we can take $\epsilon = 0.2$, $s = d\sigma_f \in \Psi(\Sigma_f)$ and $t = aca^{n-2} \in L/s$, then,

$$P_1^{-1}[P_1(st)] = \left\{ d\sigma_f a^{n-1}, \sigma_f a^{n-1}, \sigma_{uo} a^{n-1}, d\sigma_f a^{n-1-k} c a^k : 0 \leq k \leq n-1 \right\}.$$

Because $\sigma_f \notin \sigma_{uo} a^{n-1}$, the diagnosability condition function $D_1(st) = 0$. But

$$Pr(t : D_1(st) = 0 \mid t \in L/s \wedge \|t\| = n) = Pr(aca^{n-2}) = 0.21 > \epsilon. \quad (19)$$

Similarly, for the second projection $P_2 : \Sigma^* \rightarrow \Sigma_{o,2}^*$, we can take $\epsilon = 0.1$, $s = \sigma_f \in \Psi(\Sigma_f)$ and $t = aba^{n-2} \in L/s$, then,

$$P_2^{-1}[P_2(st)] = \left\{ d\sigma_f a^{n-1}, \sigma_f a^{n-1}, \sigma_{uo} a^{n-1}, \sigma_f a^{n-1-k} b a^k : 0 \leq k \leq n-1 \right\}.$$

Because $\sigma_f \notin \sigma_{uo} a^{n-1}$, the diagnosability condition function $D_2(st) = 0$. But

$$Pr(t : D_2(st) = 0 \mid t \in L/s \wedge \|t\| = n) = Pr(aba^{n-2}) = 0.16 > \epsilon. \quad (20)$$

Eqs. (19, 20) indicate that L is not diagnosable with respect to P_1 , neither is L diagnosable with respect to P_2 . \square

Proposition 2: Let $G = (Q, \Sigma, \eta, q_0)$ be a stochastic automaton, $L = L(G)$. Assume there are m local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2, \dots, m$. L is not codiagnosable with respect to $\{P_i\}$, if and only if,

$$\begin{aligned} & (\exists \epsilon > 0)(\forall n_i \in \mathbf{N})(\exists s \in \Psi(\Sigma_f) \wedge n \geq n_i)(\exists t \in L/s)(\forall j \in \{1, 2, \dots, m\}) \\ & \{Pr(t : D_j(st) = 0 \mid t \in L/s \wedge \|t\| = n) \geq \epsilon\}. \end{aligned} \quad (21)$$

Proof: It can be readily obtained from Definition 4. \square

Remark 3: If $m = 2$ and $\Sigma_f = \{\sigma_f\}$, then L being not codiagnosable means the following: there exists $\epsilon > 0$, such that for any $n_i \in \mathbf{N}$, there exist $s \in \Psi(\Sigma_f)$, $t \in L/s$, $\omega_1 \in P_1^{-1}[P_1(st)]$, and $\omega_2 \in P_2^{-1}[P_2(st)]$, satisfying $\sigma_f \notin \omega_1$, $\sigma_f \notin \omega_2$, and

$$Pr(t : D_1(st) = 0 \mid t \in L/s \wedge \|t\| = n) \geq \epsilon, \quad (22)$$

$$Pr(t : D_2(st) = 0 \mid t \in L/s \wedge \|t\| = n) \geq \epsilon. \quad (23)$$

B. Construction of Codiagnoser from a Stochastic Automaton

Firstly, we will define a logical finite state automaton from a given stochastic automaton.

Definition 5: Let $G = (Q, \Sigma, \eta, q_0)$ be a given stochastic automaton. The *deterministic finite automaton (DFA) deduced by G* is defined as $G' = (Q, \Sigma, \delta, q_0)$ with the same sets of states and events, but the partial transition function $\delta : Q \times \Sigma \rightarrow Q$ in G' is determined by probability function $\eta : Q \times \Sigma \times Q \rightarrow [0, 1]$: for any $q, q' \in Q$ and any $\sigma \in \Sigma$,

$$\delta(q, \sigma) = q' \quad \text{iff} \quad \eta(q, \sigma, q') > 0. \quad (24)$$

And δ can be extended to Σ^* in the usual manner, i.e., for any $q \in Q$, $s \in \Sigma^*$ and $\sigma \in \Sigma$,

$$\delta(q, \epsilon) = q, \quad \delta(q, s\sigma) = \delta(\delta(q, s), \sigma).$$

It can be readily verified that $L(G) = L(G')$.

We now present the construction of the codiagnoser for SDESSs, which is a DFA built on a given stochastic automaton $G = (Q, \Sigma, \eta, q_0)$ with some local observations. Without loss of generality, assume there are two local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2$. We construct the codiagnoser for SDESSs in terms of the following steps.

Step 1: Construct a diagnoser G'_D for the DFA G' deduced by G .

Let $G' = (Q, \Sigma, \delta, q_0)$ be the DFA deduced by G according to Definition 5. From the global projection $P : \Sigma^* \rightarrow \Sigma_o^*$, where $\Sigma_o = \Sigma_{o,1} \cup \Sigma_{o,2}$, we can construct a diagnoser G'_D for G' by means of the approach in [25, 26], i.e.,

$$G'_D = (Q_D, \Sigma_o, \delta_D, \chi_0), \quad (25)$$

where Q_D is the set of states of the diagnoser, $\Sigma_o = \Sigma_{o,1} \cup \Sigma_{o,2}$, δ_D is the transition function of the diagnoser, and the initial state of the diagnoser is $\chi_0 = \{(q_0, N)\} \in Q_D$.

Step 2: Construct the local stochastic diagnosers $\{G_d^i : i = 1, 2\}$ for G .

According to the projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2$, we can construct two local stochastic diagnosers G_d^1 and G_d^2 by means of the approach in [31], i.e.,

$$G_d^1 = (Q_1, \Sigma_{o,1}, \delta_1, \chi_0, \Phi_1, \phi_0), \quad (26)$$

$$G_d^2 = (Q_2, \Sigma_{o,2}, \delta_2, \chi_0, \Phi_2, \phi_0), \quad (27)$$

where Φ_1 and Φ_2 are the sets of probability transition matrices, and ϕ_0 is the initial probability mass function on χ_0 , and each element $q_i^1 \in Q_1$ or $q_i^2 \in Q_2$ is of the form

$$q_i^1 = \{(q_{i1}^1, \ell_{i1}^1), \dots, (q_{im}^1, \ell_{im}^1)\}, \quad q_i^2 = \{(q_{i1}^2, \ell_{i1}^2), \dots, (q_{in}^2, \ell_{in}^2)\}, \quad (28)$$

where $q_{ij}^1, q_{ij}^2 \in Q$ and $\ell_{ij}^1, \ell_{ij}^2 \in \Delta = \{N\} \cup 2^{\{F_1, \dots, F_k\}}$. We can refer to [31] for the details.

Step 3: Construct the codiagnoser G_T of testing the codiagnosability for G .

Although system G is a stochastic automaton, the codiagnoser that we will construct subsequently to test the codiagnosability is a DFA, which is interpreted as follows. On the one side, the local diagnosers G_d^1 and G_d^2 are stochastic, so the codiagnoser being DFA has also appended a probabilistic structure through G_d^1 and G_d^2 . On the other side, we can decrease the cost of constructing the codiagnoser since a DFA is simpler than a stochastic automaton.

The codiagnoser of testing the codiagnosability for G is constructed as a DFA

$$G_T = (Q_T, \Sigma_T, \delta_T, q_0^T), \quad (29)$$

where Q_T is the set of states of the codiagnoser, Σ_T is the set of inputting events, δ_T is the transition function, and q_0^T is initial element. More specifically, they are defined as follows:

(1) $Q_T = Q_D \times Q_1 \times Q_2$, and element $q^T = (q^D, q^1, q^2) \in Q_T$ is of the form

$$q^T = (q^D, \{(q_1^1, \ell_1^1), \dots, (q_m^1, \ell_m^1)\}, \{(q_1^2, \ell_1^2), \dots, (q_n^2, \ell_n^2)\}), \quad (30)$$

where $q^1 = \{(q_1^1, \ell_1^1), \dots, (q_m^1, \ell_m^1)\} \in Q_1$, and $q^2 = \{(q_1^2, \ell_1^2), \dots, (q_n^2, \ell_n^2)\} \in Q_2$. A triple $(q^D, (q_i^1, \ell_i^1), (q_j^2, \ell_j^2))$ is called a *component of q^T* , where $(q_i^1, \ell_i^1) \in q^1$ and $(q_j^2, \ell_j^2) \in q^2$.

(2) $\Sigma_T \subseteq \Sigma_o \times \Sigma_o \times \Sigma_o$, and $\sigma^T \in \Sigma_T$ is of the form $\sigma^T = (\sigma^D, \sigma^1, \sigma^2)$, where

$$\sigma^1 = \begin{cases} \sigma^D, & \text{if } \sigma^D \in \Sigma_{o,1}, \\ \epsilon, & \text{if } \sigma^D \notin \Sigma_{o,1}, \end{cases} \quad \sigma^2 = \begin{cases} \sigma^D, & \text{if } \sigma^D \in \Sigma_{o,2}, \\ \epsilon, & \text{if } \sigma^D \notin \Sigma_{o,2}. \end{cases} \quad (31)$$

(3) The transition function δ_T is defined as: for any $q^T = (q^D, q^1, q^2) \in Q_T$ and for any $\sigma^T = (\sigma^D, \sigma^1, \sigma^2) \in \Sigma_T$, we discuss it by the following three cases.

i) If $\sigma^D \in \Sigma_{o,1} \cap \Sigma_{o,2}$, then $\sigma^T = (\sigma^D, \sigma^D, \sigma^D)$, and

$$\begin{aligned} \delta_T(q^T, \sigma^T) &= (\delta_D(q^D, \sigma^D), \delta_1(q^1, \sigma^D), \delta_2(q^2, \sigma^D)) \\ \Leftrightarrow \quad &\delta_D(q^D, \sigma^D) \neq \emptyset, \quad \delta_1(q^1, \sigma^D) \neq \emptyset, \quad \delta_2(q^2, \sigma^D) \neq \emptyset. \end{aligned} \quad (32)$$

ii) If $\sigma^D \in \Sigma_{o,1} - \Sigma_{o,2}$, then $\sigma^T = (\sigma^D, \sigma^D, \epsilon)$, and

$$\begin{aligned} \delta_T(q^T, \sigma^T) &= (\delta_D(q^D, \sigma^D), \delta_1(q^1, \sigma^D), q^2) \\ \Leftrightarrow \quad &\delta_D(q^D, \sigma^D) \neq \emptyset, \quad \delta_1(q^1, \sigma^D) \neq \emptyset. \end{aligned} \quad (33)$$

iii) If $\sigma^D \in \Sigma_{o,2} - \Sigma_{o,1}$, then $\sigma^T = (\sigma^D, \epsilon, \sigma^D)$, and

$$\begin{aligned} \delta_T(q^T, \sigma^T) &= (\delta_D(q^D, \sigma^D), q^1, \delta_2(q^2, \sigma^D)) \\ \Leftrightarrow \quad &\delta_D(q^D, \sigma^D) \neq \emptyset, \quad \delta_2(q^2, \sigma^D) \neq \emptyset. \end{aligned} \quad (34)$$

(4) The initial element of G_T is defined as $q_0^T = \{\chi_0, \chi_0, \chi_0\} \in Q_T$, where $\chi_0 = \{(q_0, N)\}$.

This codiagnoser associated with the local stochastic diagnosers G_d^1 and G_d^2 (the part of dash line in Fig. 2) will be used to perform decentralized failure diagnosis of SDESs and to describe a necessary and sufficient condition of the codiagnosability for SDESs in Section IV.

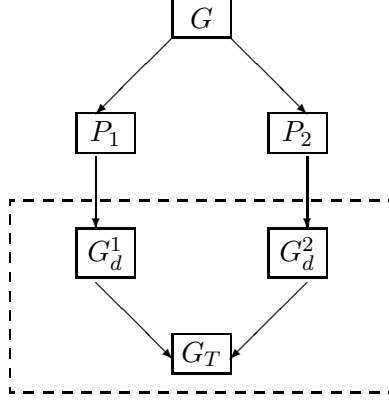


Fig. 2. The architecture of decentralized failure diagnosis of SDESs.

IV. Necessary and Sufficient Condition of Codiagnosability for SDESs

In this section, we give some properties of the codiagnoser. In particular, a necessary and sufficient condition of the codiagnosability for SDESs is presented. And we propose an approach in detail to check whether codiagnosability is violated. As well, some examples are given to illustrate the results we present.

A. Some Properties of the Codiagnoser

Firstly, we give some basic properties of the codiagnosers as follows.

Proposition 3: Let $G = (Q, \Sigma, \eta, q_0)$ be a stochastic automaton with two projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$ (where $i = 1, 2$). $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ is the codiagnoser of G . For any $q^T = (q^D, q^1, q^2) \in Q_T$, there exists $s^T = (s^D, s^1, s^2) \in \Sigma_T^*$ such that

$$\delta_D(\chi_0, s^D) = q^D, \quad \delta_1(\chi_0, s^1) = q^1, \quad \delta_2(\chi_0, s^2) = q^2, \quad (35)$$

$$P_1(s^D) = P_1(s^1), \quad P_2(s^D) = P_2(s^2), \quad (36)$$

where $s^D \in \Sigma_o^*$, $s^1 \in \Sigma_{o,1}^*$, and $s^2 \in \Sigma_{o,2}^*$.

Proof: From the above construction of the codiagnoser, we know that for any $q^T = (q^D, q^1, q^2) \in Q_T$, there exists $s^T \in \Sigma_T^*$ such that $\delta_T(q_0^T, s^T) = q^T$. We prove the proposition by induction on $\|s^T\|$, the length of s^T .

Basis: If $\|s^T\| = 1$, then from (31) we have $s^T = (\sigma^D, \sigma^D, \sigma^D)$, or $s^T = (\sigma^D, \sigma^D, \epsilon)$, or $s^T = (\sigma^D, \epsilon, \sigma^D)$. It is clear that Eqs. (35, 36) hold.

Induction: Let $\delta_T(q_1^T, \sigma^T) = q^T$ where $q_1^T = (q_1^D, q_1^1, q_1^2) \in Q_T$. By the assumption of induction there exists $s^T \in \Sigma_T^*$, where $\|s^T\| = n$, such that $\delta_T(q_0^T, s^T) = q_1^T$, and Eqs. (35, 36) hold. There are three cases to be considered for $s^T \sigma^T$, where $\sigma^T = (\sigma^D, \sigma^1, \sigma^2)$.

Case 1: If $\sigma^D \in \Sigma_{o,1} \cap \Sigma_{o,2}$, then $\sigma^D = \sigma^1 = \sigma^2$. Therefore,

$$\delta_T(q_0^T, s^T \sigma^T) = \delta_T(\delta_T(q_0^T, s^T), \sigma^T) = \delta_T(q_1^T, \sigma^T) = q^T.$$

That is,

$$\begin{aligned} \delta_D(\chi_0, s^D \sigma^D) &= q^D, & \delta_1(\chi_0, s^1 \sigma^1) &= q^1, & \delta_2(\chi_0, s^2 \sigma^2) &= q^2, \\ P_1(s^D \sigma^D) &= P_1(s^D)P_1(\sigma^D) = P_1(s^1)P_1(\sigma^1) = P_1(s^1 \sigma^1). \end{aligned}$$

Similarly, we have $P_2(s^D \sigma^D) = P_2(s^2 \sigma^2)$.

Case 2: If $\sigma^D \in \Sigma_{o,1} - \Sigma_{o,2}$, then $\sigma^D = \sigma^1$, but $\sigma^2 = \epsilon$. Therefore,

$$\delta_1(\chi_0, s^1 \sigma^1) = \delta_1(\delta_1(\chi_0, s^1), \sigma^1) = \delta_1(q_1^1, \sigma^1) = q^1,$$

but $\delta_2(\chi_0, s^2 \sigma^2) = \delta_2(\chi_0, s^2) = q_1^2 = q^2$. We also have $P_1(s^D \sigma^D) = P_1(s^1 \sigma^1)$ for the same reason as in Case 1, and

$$P_2(s^D \sigma^D) = P_2(s^D)P_2(\sigma^D) = P_2(s^2) \epsilon = P_2(s^2 \sigma^2).$$

Case 3: If $\sigma^D \in \Sigma_{o,2} - \Sigma_{o,1}$, then $\sigma^D = \sigma^2$, but $\sigma^1 = \epsilon$, we can similarly verify that Eqs. (35, 36) hold for $s^T \sigma^T$. \square

Proposition 4: Let $G = (Q, \Sigma, \eta, q_0)$ be a stochastic automaton with two projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$ (where $i = 1, 2$) and let $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ be the codiagnoser of G , and $q^T = (q^D, q^1, q^2) \in Q_T$.

(1) If $(q_a^1, \ell_a^1), (q_b^1, \ell_b^1) \in q^1$, $F \in \ell_a^1$ but $F \notin \ell_b^1$, then there exist $\omega_1, \omega_2 \in \Sigma_{o,1}^*$ such that $\delta(q_0, \omega_1) = q_a^1$, $\delta(q_0, \omega_2) = q_b^1$, $\sigma_f \in \omega_1$, $\sigma_f \notin \omega_2$, and $P_1(\omega_1) = P_1(\omega_2)$.

(2) If $(q_a^2, \ell_a^2), (q_b^2, \ell_b^2) \in q^2$, $F \in \ell_a^2$ but $F \notin \ell_b^2$, then there exist $v_1, v_2 \in \Sigma_{o,2}^*$ such that $\delta(q_0, v_1) = q_a^2$, $\delta(q_0, v_2) = q_b^2$, $\sigma_f \in v_1$, $\sigma_f \notin v_2$, and $P_2(v_1) = P_2(v_2)$.

Proof: (1) Let $(q_a^1, \ell_a^1), (q_b^1, \ell_b^1) \in q^1$, $F \in \ell_a^1$ but $F \notin \ell_b^1$. Since every component in each element of Q_1 is reachable from the initial state q_0 . Therefore, there exist $\omega_1, \omega_2 \in \Sigma_{o,1}^*$ such that $\delta(q_0, \omega_1) = q_a^1$ and $\delta(q_0, \omega_2) = q_b^1$. By the definition of label propagation function [31], we have $\sigma_f \in \omega_1$ and $\sigma_f \notin \omega_2$, because $F \in \ell_a^1$ and $F \notin \ell_b^1$. From the construction of diagnoser G_d^1 , due to both (q_a^1, ℓ_a^1) and (q_b^1, ℓ_b^1) in q^1 , we know that ω_1 and ω_2 have the same strings filtered by projection P_1 . That is, $P_1(\omega_1) = P_1(\omega_2)$.

(2) It can be proved similarly. \square

B. Necessary and Sufficient Condition of Codiagnosability for SDESs

In this subsection, we will present the necessary and sufficient condition of codiagnosability for SDESs.

Definition 6: Let $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ be a codiagnoser of a stochastic automaton $G = (Q, \Sigma, \eta, q_0)$. A set $\{q_1^T, \sigma_1^T, q_2^T, \sigma_2^T, \dots, q_k^T, \sigma_k^T, q_1^T\}$ is said to form a *cycle* in G_T , if

$$\delta_T(q_j^T, \sigma_j^T) = q_{j+1}^T, \quad \delta_T(q_k^T, \sigma_k^T) = q_1^T,$$

where $q_1^T, q_2^T, \dots, q_k^T \in Q_T$, $\sigma_1^T, \sigma_2^T, \dots, \sigma_k^T \in \Sigma_T$, and $j = 1, 2, \dots, k-1$.

Definition 7: Let $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ be a codiagnoser of a stochastic automaton $G = (Q, \Sigma, \eta, q_0)$, and $q^T = (q^D, q^1, q^2) \in Q_T$.

(1) If both q^1 and q^2 are F -certain in diagnoser G_d^1 and diagnoser G_d^2 , respectively, then q^T is called to be F -certain in G_T .

(2) If both q^1 and q^2 are F -uncertain in G_d^1 and G_d^2 , respectively, then q^T is called to be F -uncertain in G_T .

For example, in Fig. 9, the states $(\{(1, N)\}, \{(0, N)\}, \{(1, N)\})$ and $(\{(6, F)\}, \{(6, F)\}, \{(5, F), (6, F)\})$ are F -certain in G_T , the state

$$(\{(2, F), (3, F), (4, N)\}, \{(2, F), (3, F), (4, N), (5, F)\}, \{(2, F), (3, F), (4, N)\})$$

is F -uncertain in G_T , but $(\{(5, F)\}, \{(2, F), (3, F), (4, N), (5, F)\}, \{(5, F), (6, F)\})$ is neither F -certain nor F -uncertain in G_T .

Definition 8: Let $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ be a codiagnoser of a stochastic automaton $G = (Q, \Sigma, \eta, q_0)$. Let $q^T = (q^D, q^1, q^2) \in Q_T$, and, let $(q^D, (q_a^1, \ell_a^1), (q_b^2, \ell_b^2))$ be a component of q^T . If both (q^1, q_a^1, ℓ_a^1) and (q^2, q_b^2, ℓ_b^2) are recurrent components of q^1 in G_d^1 and q^2 in G_d^2 , respectively, then $(q^D, (q_a^1, \ell_a^1), (q_b^2, \ell_b^2))$ is called a *recurrent component of q^T in G_T* . Furthermore, if the recurrent component $(q^D, (q_a^1, \ell_a^1), (q_b^2, \ell_b^2))$ satisfies $F \in \ell_a^1$ and $F \in \ell_b^2$, then it is called a *recurrent component bearing the label F* .

Definition 9: Let $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ be a codiagnoser of a stochastic automaton $G = (Q, \Sigma, \eta, q_0)$. Let $q^T = (q^D, q^1, q^2) \in Q_T$, and, let $(q^D, (q_a^1, \ell_a^1), (q_b^2, \ell_b^2))$ be a component of q^T . If $q_a^1 = q_b^2$, $\ell_a^1 = \ell_b^2$, and there exists $\omega \in L$ such that

$$\delta(q_0, \omega) = q_a^1, \quad \delta_1(\chi_0, P_1(\omega)) = q^1, \quad \delta_2(\chi_0, P_2(\omega)) = q^2, \quad (37)$$

then the component $(q^D, (q_a^1, \ell_a^1), (q_b^2, \ell_b^2))$ is called *uniform*.

For example, in Fig. 5, $(\{(3, F)\}, (3, F), (3, F))$ is a uniform component of state q^T in codiagnoser G_T , where $q^T = (\{(3, F)\}, \{(2, F), (3, F), (4, N), (5, F)\}, \{(3, F)\})$. In fact, we can take $\omega = d\sigma_fca \in L$ which satisfies (37).

Proposition 5: Let $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ be a codiagnoser of stochastic automaton $G = (Q, \Sigma, \eta, q_0)$ with two projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2$. If L is not codiagnosable with respect to $\{P_i : i = 1, 2\}$, then there exists an F -uncertain state with a recurrent component bearing the label F in G_T .

Proof: Since L is not codiagnosable, by Proposition 2, there exists $\epsilon > 0$, and for any $n_i \in \mathbf{N}$, there exist $s \in \Psi(\Sigma_f)$ and $t \in L/s$ (where $n \geq n_i$), such that

$$Pr(t : D_1(st) = 0 \mid t \in L/s \wedge \|t\| = n) \geq \epsilon, \quad (38)$$

$$Pr(t : D_2(st) = 0 \mid t \in L/s \wedge \|t\| = n) \geq \epsilon. \quad (39)$$

Denote by $\delta_D(\chi_0, P(st)) = q^D$, $\delta_1(\chi_0, P_1(st)) = q^1$, $\delta_2(\chi_0, P_2(st)) = q^2$. We assert that $(q^D, q^1, q^2) \in Q_T$ is an F -uncertain state in G_T . Otherwise, if $q^1 \in Q_1$ is an F -certain state in G_d^1 , then for any $\omega \in P_1^{-1}[P_1(st)]$, we have $\sigma_f \in \omega$. That is, $D_1(st) = 1$ always holds. Therefore,

$$Pr(t : D_1(st) = 0 \mid t \in L/s \wedge \|t\| = n) = 0,$$

which is in contradiction with (38). So q^1 is an F -uncertain state in G_d^1 . Similarly, from (39) we can know that q^2 is an F -uncertain state in G_d^2 . Hence, $(q^D, q^1, q^2) \in Q_T$ is an F -uncertain state in G_T . Furthermore, by Lemma 1 and (38, 39), both q^1 and q^2 contain respectively a recurrent component bearing the label F . That is, $(q^D, q^1, q^2) \in Q_T$ is an F -uncertain state in G_T with a recurrent component bearing the label F . \square

Using the above results, we present a necessary and sufficient condition of the codiagnosability for SDESs as follows.

Theorem 1: Let $G = (Q, \Sigma, \eta, q_0)$ be a stochastic automaton with two local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2$. Let $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ be a codiagnoser of G . Then $L = L(G)$ is not codiagnosable with respect to $\{P_i\}$, if and only if, there exists a cycle $C^T = \{q_k^T, \sigma_k^T, q_{k+1}^T, \sigma_{k+1}^T, \dots, q_{k+h}^T, \sigma_{k+h}^T, q_k^T\}$ in G_T such that each state q_{k+i}^T with a uniform recurrent component bearing the label F is F -uncertain in G_T , where $i \in [0, h]$.

Proof: Sufficiency: Assume that there exists a cycle

$$C^T = \{q_k^T, \sigma_k^T, q_{k+1}^T, \sigma_{k+1}^T, \dots, q_{k+h}^T, \sigma_{k+h}^T, q_k^T\}$$

in G_T , such that each state $q_{k+i}^T = (q_{k+i}^D, q_{k+i}^1, q_{k+i}^2)$ with a uniform recurrent component (denoted by $(q_{k+i}^D, (q_a, \ell_a), (q_a, \ell_a))$) bearing the label F is F -uncertain in G_T , where $i \in [0, h]$. By Definition 9, there exists $\omega \in L$ such that $\sigma_f \in \omega$ and

$$\delta(q_0, \omega) = q_a, \quad \delta_1(\chi_0, P_1(\omega)) = q_{k+i}^1, \quad \delta_2(\chi_0, P_2(\omega)) = q_{k+i}^2. \quad (40)$$

Since q_{k+i}^T is a state of the cycle C^T , there is a path ending with the cycle C^T in G_T :

$$path = \sigma_0^T \sigma_1^T \dots (\sigma_k^T \dots \sigma_{k+i}^T \dots \sigma_{k+h}^T)^n. \quad (41)$$

We take $s \in \overline{\omega}$, $t \in L/s$ and $u \in L/st$ such that $s \in \Psi(\Sigma_f)$, $st = \omega$ and

$$P_1(stu) = \sigma_0^1 \sigma_1^1 \cdots (\sigma_k^1 \cdots \sigma_{k+i}^1 \cdots \sigma_{k+h}^1)^n,$$

$$P_2(stu) = \sigma_0^2 \sigma_1^2 \cdots (\sigma_k^2 \cdots \sigma_{k+i}^2 \cdots \sigma_{k+h}^2)^n.$$

Notice that q_{k+i}^T is F -uncertain in G_T , there exist $\omega_1 \in P_1^{-1}[P_1(st)]$ and $\omega_2 \in P_2^{-1}[P_2(st)]$ such that $\sigma_f \notin \omega_1$ and $\sigma_f \notin \omega_2$, i.e., $D_1(st) = 0$ and $D_2(st) = 0$. That is,

$$Pr(u : D_1(stu) = 0 \mid tu \in L/s \wedge \|tu\| = n) = 1, \quad (42)$$

$$Pr(u : D_2(stu) = 0 \mid tu \in L/s \wedge \|tu\| = n) = 1. \quad (43)$$

Furthermore, $Pr(t : t \in L/s) > 0$ since $t \in L/s$. Let $0 < \epsilon < Pr(t : t \in L/s)$, then by (42, 43) we have

$$\begin{aligned} & Pr(tu : D_1(stu) = 0 \mid tu \in L/s \wedge \|tu\| = n) \\ &= Pr(t : t \in L/s) Pr(u : D_1(stu) = 0 \mid tu \in L/s \wedge \|tu\| = n) > \epsilon, \end{aligned} \quad (44)$$

and

$$\begin{aligned} & Pr(tu : D_2(stu) = 0 \mid tu \in L/s \wedge \|tu\| = n) \\ &= Pr(t : t \in L/s) Pr(u : D_2(stu) = 0 \mid tu \in L/s \wedge \|tu\| = n) > \epsilon. \end{aligned} \quad (45)$$

By Remark 3, we obtain that L is not codiagnosable with respect to $\{P_i : i = 1, 2\}$.

Necessity: Assume that L is not codiagnosable. By Proposition 2 and Remark 3, there is $\epsilon > 0$, such that for any $n_i \in \mathbf{N}$, there exist $s \in \Psi(\Sigma_{f_i})$ and $t \in L/s$, where $\|t\| = n \geq n_i$, satisfying Ineqs. (22, 23). Therefore, there exist $\omega_1 \in P_1^{-1}[P_1(st)]$ and $\omega_2 \in P_2^{-1}[P_2(st)]$ such that $\sigma_f \notin \omega_1$ and $\sigma_f \notin \omega_2$. Let $\|Q\|$ be the number of states of G . If we take n_i big enough such that $\|st\| > \|Q\|$, then there will be a cycle C in G along string st . According to Assumption (A2), the cycle C must contain an observable event in Σ_o . Therefore, there exists a cycle C^D in diagnoser G'_D corresponding to the cycle C , denoted by

$$C^D = \{q_k^D, \sigma_k^D, q_{k+1}^D, \sigma_{k+1}^D, \dots, q_{k+h}^D, \sigma_{k+h}^D, q_k^D\}.$$

We denote the path in G'_D ending with the cycle C^D as

$$path = \sigma_0^D \sigma_1^D \cdots (\sigma_k^D \cdots \sigma_{k+i}^D \cdots \sigma_{k+h}^D)^n.$$

According to the construction of the codiagnoser G_T , we can obtain a cycle in G_T :

$$C^T = \{q_k^T, \sigma_k^T, q_{k+1}^T, \sigma_{k+1}^T, \dots, q_{k+h}^T, \sigma_{k+h}^T, q_k^T\}, \quad (46)$$

where $\sigma_j^T = (\sigma_j^D, \sigma_j^1, \sigma_j^2)$, $j \in [k, k+h]$. Denote

$$q_{i_0}^1 = \delta_1(\chi_0, P_1(st)), \quad q_{i_0}^2 = \delta_2(\chi_0, P_2(st)).$$

Notice that st satisfies Ineq. (22, 23). By Lemma 1, both $q_{i_0}^1$ and $q_{i_0}^2$ contain respectively a recurrent component bearing the label F . Furthermore, from $\sigma_f \in s$, $\sigma_f \notin \omega_1$ and $\sigma_f \notin \omega_2$, we know that both $q_{i_0}^1$ and $q_{i_0}^2$ are F -uncertain states in G_d^1 and in G_d^2 , respectively. That is, $q_{i_0}^T = (q_{i_0}^D, q_{i_0}^1, q_{i_0}^2) \in Q_T$ is an F -uncertain state in G_T .

In the following we will verify that each state $q_j^T = (q_j^D, q_j^1, q_j^2)$ with a uniform recurrent component (denoted by $(q_j^D, (q_a, \ell_a), (q_a, \ell_a))$) bearing the label F in the cycle C^T is F -uncertain in G_T , where $q_j^T \in Q_T$ and $j \in [k, k+h]$.

Case 1: If $q_j^T = q_{i_0}^T$, then it has been verified above that $q_{i_0}^T$ is F -uncertain in G_T .

Case 2: Assume $q_j^T \neq q_{i_0}^T$. Due to both $q_{i_0}^T$ and q_j^T being states in the cycle C^T , there exists a component of q_j^1 reachable from the recurrent component of $q_{i_0}^1$, and there exists a component of q_j^2 reachable from the recurrent component of $q_{i_0}^2$. Likewise, there exists a component of $q_{i_0}^1$ reachable from the recurrent component (q_j^1, q_a, ℓ_a) , and there exists a component of $q_{i_0}^2$ reachable from the recurrent component (q_j^2, q_a, ℓ_a) . By Lemma 2, both q_j^1 and q_j^2 are F -uncertain in G_d^1 and in G_d^2 , respectively, since $q_{i_0}^T$ is F -uncertain in G_T . Therefore, $q_j^T = (q_j^D, q_j^1, q_j^2) \in Q_T$ is an F -uncertain state in G_T . \square

Remark 4: From the proof of Theorem 1, we know that if there is only one projection in system G , then Theorem 1 degenerates to Theorem 3 of [31]. Therefore, the centralized failure diagnosis in [31] can be regarded as a special case of the decentralized failure diagnosis here.

C. The Computing Process of Checking the Codiagnosability in SDESSs

Let $G = (Q, \Sigma, \eta, q_0)$ be a stochastic automaton with two projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2$. G' is a DFA deduced by G . We give a computing process to check whether the codiagnosability condition of Theorem 1 is violated.

Step 1: Construct the diagnoser G'_D (i.e., Eq. (25)) for the DFA G' and the local stochastic diagnosers $\{G_d^i : i = 1, 2\}$ (i.e., Eqs. (26, 27)) for G .

This specific procedure can be seen in Part B of Section III for the details.

Step 2: Construct the codiagnoser $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ (i.e., Eq. (29)) for G .

Also, this specific procedure can be seen in Part B of Section III for the details.

Step 3: Check whether there exists a cycle in the codiagnoser G_T .

If there does not exist a cycle in the codiagnoser G_T , then L is codiagnosable with respect to $\{P_i\}$. Otherwise, we perform the next step.

Step 4: Check whether the states in each cycle satisfy the following condition: each state with a uniform recurrent component bearing the label F is F -uncertain in G_T .

If each state with a uniform recurrent component bearing the label F in each cycle is F -uncertain in G_T , then we further perform the next step. Otherwise, L is codiagnosable.

Step 5: Check whether the codiagnosability is violated.

If there exists a cycle C^T whose each state q^T with a uniform recurrent component bearing the label F is F -uncertain, then L is not codiagnosable. Otherwise, L is codiagnosable.

D. Examples of Codiagnosability for SDESs

In this subsection, we will give some examples to illustrate the applications of the necessary and sufficient condition for the codiagnosability of FDESs and its computing method we presented above.

Example 3. Consider the stochastic automaton G as in Example 1. $L = L(G)$. Assume there are two projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, where $i = 1, 2$ and $\Sigma_{o,1} = \{a, b\}$, $\Sigma_{o,2} = \{a, c\}$.

From Example 2, we know that L is neither diagnosable with respect to P_1 , nor diagnosable with respect to P_2 . But Example 1 shows that L is codiagnosable. In the following we use Theorem 1 and the above computing process to test these results.

According to the global projection $P : \Sigma^* \rightarrow \Sigma_o^*$ where $\Sigma_o = \Sigma_{o,1} \cup \Sigma_{o,2} = \{a, b, c\}$, and the projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, (where $i = 1, 2$), we can construct the diagnoser $G'_D = (Q_D, \Sigma_o, \delta_D, \chi_0)$ for DFA G' as in Fig. 3, and two local stochastic diagnosers G_d^1, G_d^2 as in Fig. 4, where $G_d^1 = (Q_1, \Sigma_{o,1}, \delta_1, \chi_0, \Phi_1, \phi_0)$, $G_d^2 = (Q_2, \Sigma_{o,2}, \delta_2, \chi_0, \Phi_2, \phi_0)$.

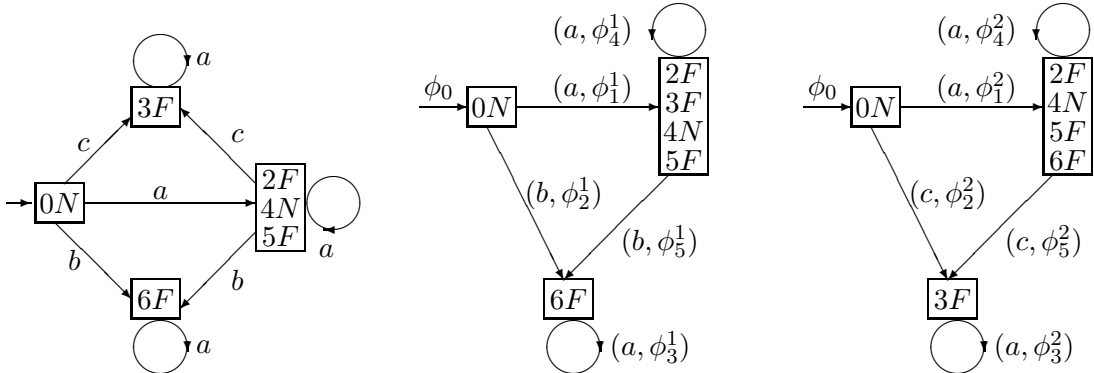


Fig. 3. Diagnoser G'_D in Example 3. Fig. 4. Local diagnosers G_d^1 (left one) and G_d^2 (right one) in Example 3.

For the local stochastic diagnoser G_d^1 , the set of probability transition matrices $\Phi_1 = \{\phi_0, \phi_1^1, \phi_2^1, \phi_3^1, \phi_4^1, \phi_5^1\}$, where $\phi_0 = [1]$, $\phi_1^1 = [0.35, 0.15, 0.2, 0.24]$, $\phi_2^1 = [0.06]$, $\phi_3^1 = [1]$,

$$\phi_4^1 = \begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix}, \quad \phi_5^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix}.$$

Therefore, the recurrent components bearing F are $(q_2^1, 6, F)$ and $(q_3^1, 3, F)$, where $q_2^1 = \{(6, F)\}$ and $q_3^1 = \{(2, F), (3, F), (4, N), (5, F)\}$. Notice that q_3^1 is not F -certain in G_d^1 , so, neither is L diagnosable with respect to P_1 by Lemma 3.

For the local stochastic diagnoser G_d^2 , the set of probability transition matrices $\Phi_2 = \{\phi_0, \phi_1^2, \phi_2^2, \phi_3^2, \phi_4^2, \phi_5^2\}$, where $\phi_0 = [1]$, $\phi_1^2 = [0.35, 0.2, 0.24, 0.06]$, $\phi_2^2 = [0.15]$, $\phi_3^2 = [1]$,

$$\phi_4^2 = \begin{bmatrix} 0.7 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \phi_5^2 = \begin{bmatrix} 0.3 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Therefore, the recurrent component bearing F are $(q_2^2, 3, F)$ and $(q_3^2, 6, F)$, where $q_2^2 = \{(3, F)\}$ and $q_3^2 = \{(2, F), (4, N), (5, F), (6, F)\}$. Notice that q_3^2 is not F -certain in G_d^2 , so L is not diagnosable with respect to P_2 by Lemma 3, either.

Now we construct the codiagnoser G_T to test the codiagnosability for G . The codiagnoser is a DFA $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ as in Fig. 5, where $\Sigma_T = \{(a, a, a), (b, b, \epsilon), (c, \epsilon, c)\}$.

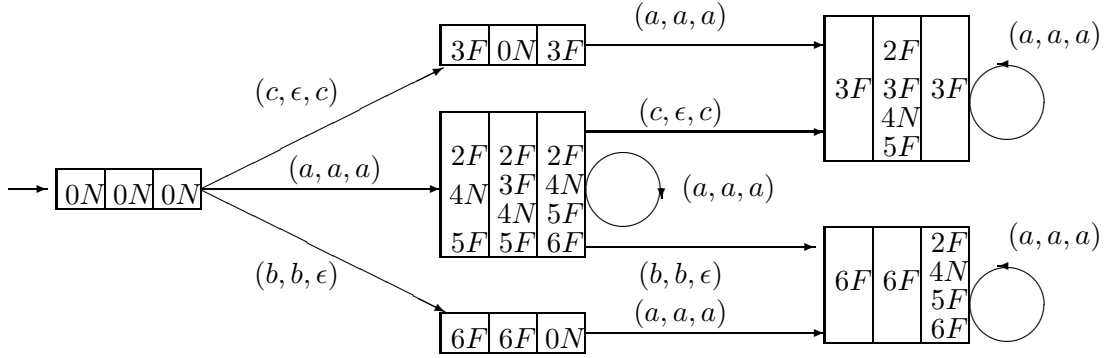


Fig. 5. Codiagnoser G^T in Example 3.

From Fig. 5, we know that there are three cycles C_1^T, C_2^T, C_3^T in G^T as follows:

$$C_1^T = \{q_2^T, (a, a, a), q_2^T\}, \quad C_2^T = \{q_3^T, (a, a, a), q_3^T\}, \quad C_3^T = \{q_5^T, (a, a, a), q_5^T\}, \quad (47)$$

where

$$\begin{aligned} q_2^T &= (\{(3, F)\}, \{(2, F), (3, F), (4, N), (5, F)\}, \{(3, F)\}), \\ q_3^T &= (\{(2, F), (4, N), (5, F)\}, \{(2, F), (3, F), (4, N), (5, F)\}, \{(2, F), (4, N), (5, F), (6, F)\}), \\ q_5^T &= (\{(6, F)\}, \{(6, F)\}, \{(2, F), (4, N), (5, F), (6, F)\}). \end{aligned}$$

In cycle C_1^T , state q_2^T contains a uniform recurrent component $(\{(3, F)\}, (3, F), (3, F))$ bearing the label F , but it is not F -uncertain in G_T . In cycle C_2^T , state q_3^T is an F -uncertain state of G_T , but it does not contain any uniform recurrent component. Likewise, in cycle C_3^T ,

state q_5^T contains a uniform recurrent component $(\{(6, F)\}, (6, F), (6, F))$ bearing the label F , but it is not F -uncertain in G_T . Therefore, there does not exist the cycle whose each state with a uniform recurrent component bearing the label F is F -uncertain. By Theorem 1, we know that L is codiagnosable, which coincides with the result of Example 1. \square

Example 4. Consider the stochastic automaton $G = (Q, \Sigma, \eta, q_0)$ represented by Fig.6, where $Q = \{q_0, \dots, q_6\}$, $\Sigma = \{a, b, c, d, \sigma_{uo}, \sigma_f\}$, and $\Sigma_f = \{\sigma_f\}$. $L = L(G)$. Assume there are two projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, (where $i = 1, 2$), $\Sigma_{o,1} = \{a, b\}$, $\Sigma_{o,2} = \{a, d\}$.

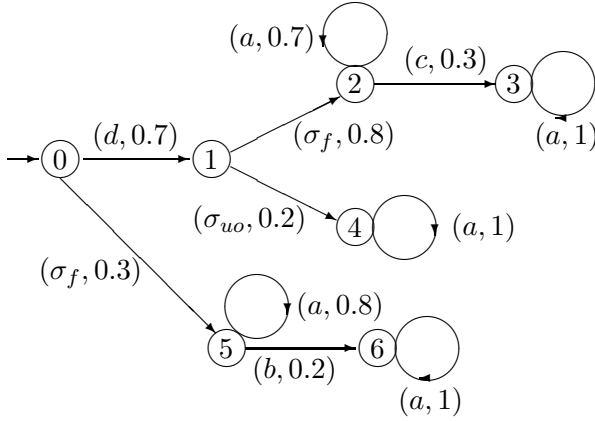


Fig. 6. Stochastic automaton of Example 4.

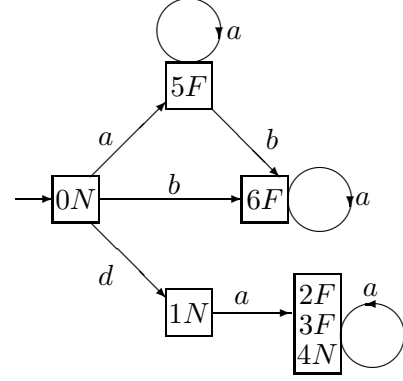


Fig. 7. Diagnoser G'_D in Example 4.

We assert that L is not codiagnosable with respect to $\{P_i : i = 1, 2\}$. In fact, we can verify this conclusion by two scenarios as follows.

On the one side, we use the definition of codiagnosability of SDESs (i.e., Definition 4) to interpret L to be not F -codiagnosable with respect to $\{P_i : i = 1, 2\}$. We take $\epsilon = 0.2$, $s = d\sigma_f \in \Psi(\Sigma_f)$, and $t = aca^{n-2} \in L/s$, then

$$P_1^{-1}[P_1(st)] = \{d\sigma_{uo}a^{n-1}, \sigma_fa^{n-1}, d\sigma_fa^{n-1}, d\sigma_fa^{n-k-1}ca^k : 0 \leq k \leq n-1\},$$

$$P_2^{-1}[P_2(st)] = \{d\sigma_{uo}a^{n-1}, d\sigma_fa^{n-1}, d\sigma_fa^{n-k-1}ca^k : 0 \leq k \leq n-1\}.$$

Notice that $\sigma_f \notin d\sigma_{uo}a^{n-1}$, so $D_1(st) = 0$ and $D_2(st) = 0$. However,

$$Pr(t : D_1(st) = 0 \mid t \in L/s \wedge \|t\| = n) = Pr(aca^{n-2}) = 0.7 \times 0.3 = 0.21 > \epsilon, \quad (48)$$

$$Pr(t : D_2(st) = 0 \mid t \in L/s \wedge \|t\| = n) = Pr(aca^{n-2}) = 0.7 \times 0.3 = 0.21 > \epsilon. \quad (49)$$

Therefore, L is neither diagnosable with respect to P_1 nor diagnosable with respect to P_2 . By Definition 4 or Proposition 2, we obtain that L is not codiagnosable.

On the other side, we also can use Theorem 1 to verify that L is not codiagnosable.

According to the global projection $P : \Sigma^* \rightarrow \Sigma_o^*$ and the local projections $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$, ($i = 1, 2$), we can construct the diagnoser $G'_D = (Q_D, \Sigma_o, \delta_D, \chi_0)$ for DFA G' as Fig. 7, and

two local stochastic diagnosers G_d^1, G_d^2 as Fig. 8, where $\Sigma_o = \Sigma_{o,1} \cup \Sigma_{o,2} = \{a, b, d\}$, and

$$G_d^1 = (Q_1, \Sigma_{o,1}, \delta_1, \chi_0, \Phi_1, \phi_0), \quad G_d^2 = (Q_2, \Sigma_{o,2}, \delta_2, \chi_0, \Phi_2, \phi_0).$$

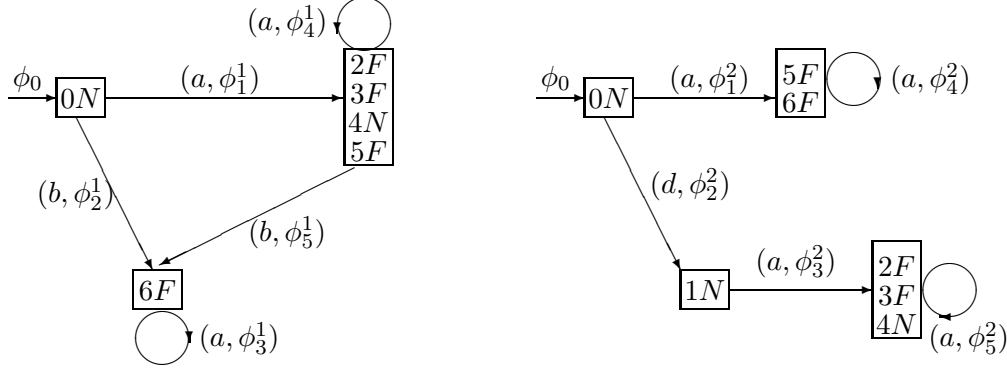


Fig. 8. Local diagnosers G_d^1 (left one) and G_d^2 (right one) in Example 4.

For the local stochastic diagnoser G_d^1 , the set of probability transition matrices $\Phi_1 = \{\phi_0, \phi_1^1, \phi_2^1, \phi_3^1, \phi_4^1, \phi_5^1\}$, where $\phi_0 = [1]$, $\phi_1^1 = [0.392, 0.168, 0.14, 0.24]$, $\phi_2^1 = [0.06]$, $\phi_3^1 = [1]$,

$$\phi_4^1 = \begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix}, \quad \phi_5^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix}.$$

Therefore, the recurrent component bearing F are $(q_2^1, 6, F)$ and $(q_3^1, 3, F)$, where $q_2^1 = \{(6, F)\}$ and $q_3^1 = \{(2, F), (3, F), (4, N), (5, F)\}$. Notice that q_3^1 is not an F -certain state of G_d^1 , so L is not diagnosable with respect to P_1 by Lemma 3.

For the local stochastic diagnoser G_d^2 , the set of probability transition matrices $\Phi_2 = \{\phi_0, \phi_1^2, \phi_2^2, \phi_3^2, \phi_4^2, \phi_5^2\}$ where $\phi_0 = [1]$, $\phi_1^2 = [0.24, 0.06]$, $\phi_2^2 = [0.7]$, $\phi_3^2 = [0.56, 0.24, 0.2]$,

$$\phi_4^2 = \begin{bmatrix} 0.8 & 0.2 \\ 0 & 1 \end{bmatrix}, \quad \phi_5^2 = \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Therefore, the recurrent component bearing F are $(q_2^2, 6, F)$ and $(q_4^2, 3, F)$, where $q_2^2 = \{(5, F), (6, F)\}$ and $q_4^2 = \{(2, F), (3, F), (4, N)\}$. Notice that q_4^2 is not F -certain in G_d^2 , so L is not diagnosable with respect to P_2 by Lemma 3, either.

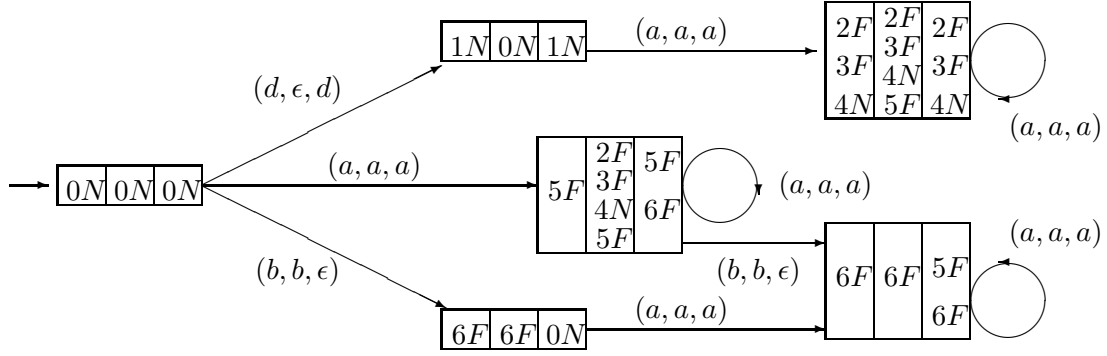


Fig. 9. Codiagnoser G^T in Example 4.

Now we construct the codiagnoser G_T to verify that L is also not codiagnosable. The codiagnoser is a DFA $G_T = (Q_T, \Sigma_T, \delta_T, q_0^T)$ as Fig. 9, where $\Sigma_T = \{(a, a, a), (b, b, \epsilon), (d, \epsilon, d)\}$.

From Fig. 9, we know that there are three cycles C_1^T, C_2^T, C_3^T in G^T as follows:

$$C_1^T = \{q_2^T, (a, a, a), q_2^T\}, \quad C_2^T = \{q_3^T, (a, a, a), q_3^T\}, \quad C_3^T = \{q_5^T, (a, a, a), q_5^T\}, \quad (50)$$

where

$$\begin{aligned} q_2^T &= (\{(2, F), (3, F), (4, N)\}, \{(2, F), (3, F), (4, N), (5, F)\}, \{(2, F), (3, F), (4, N)\}), \\ q_3^T &= (\{(5, F)\}, \{(2, F), (3, F), (4, N), (5, F)\}, \{(5, F), (6, F)\}), \\ q_5^T &= (\{(6, F)\}, \{(6, F)\}, \{(5, F), (6, F)\}). \end{aligned}$$

In the cycle C_1^T , there is only one state q_2^T and it contains a uniform recurrent component

$$(\{(2, F), (3, F), (4, N)\}, (3, F), (3, F))$$

bearing the label F . Furthermore, q_2^T is an F -uncertain state of G_T . Therefore, there does exist a cycle (i.e., C_1^T) whose each state with a uniform recurrent component bearing the label F is F -uncertain. By Theorem 1, we obtain that L is not codiagnosable. \square

V. Concluding Remarks

Recently, J. Lunze and J. Schröder [16], D. Thorsley and D. Teneketzis [31] generalized the diagnosability of classical DESs [25, 26] to the setting of stochastic DESs (SDESs). In [16], the diagnostic problem was transformed into an observation problem, and the diagnosability was obtained by an extension of an observation algorithm. In [31], the notions of A- and AA-diagnosability for stochastic automata were defined, which were weaker than those for classical automata introduced by Sampath *et al* [25, 26], and they [31] presented a necessary and sufficient condition for the diagnosability of SDESs.

However, the failure diagnosis they considered in [16, 31] was still centralized. In this paper, we have dealt with the decentralized failure diagnosis for SDESs. The centralized failure diagnosis of SDESs in [16, 31] can be viewed as a special case of the decentralized failure diagnosis presented in this paper with only one projection. We formalized the approach to decentralized failure diagnosis by introducing the notion of codiagnosability. By constructing a codiagnoser from a given stochastic automaton with multiple projections, we used the codiagnoser associated with the local diagnosers to test codiagnosability condition of SDESs. As well, a number of basic properties of the codiagnoser has been investigated. In particular, a necessary and sufficient condition for the codiagnosability of SDESs was presented, which generalizes the result of classical DESs dealt with by W. Qiu and R. Kumar [21]. Furthermore, we gave a computing method in detail to check whether codiagnosability is violated. Finally, some examples were described to illustrate the applications of the codiagnosability and its computing method.

The problem of decentralized diagnosis can be considered as one special case of distributed diagnosis in [7]. Therefore, the potential of applications of the results in this paper may be in failure diagnosis of many large complex systems which are physically distributed [7, 21, 22, 28]. Moreover, with the results obtained in this paper, a further issue worthy of consideration is the strong codiagnosability of SDESs, as the strong codiagnosability of classical DESs [21]. Another important issue is how to compute the bound in the delay of decentralized diagnosis for SDESs. We would like to consider them in subsequent work.

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